Asymptotic Tracking of Systems with Non-Symmetrical Input Deadzone Nonlinearity

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Abstract

This paper presents an adaptive control scheme for systems with uncertain asymmetrical deadzone nonlinearity at the input of a linear plant. An adaptive inverse block has been developed and used in conjunction with any conventional controllers in order to reduce the effect of deadzone nonlinearity. The deadzone inverse model is non-symmetric and implemented in continuous time. The adaptive deadzone inverse controller is smoothly differentiable and can easily be combined with any of the advanced control methodologies. The asymptotic stability of the closed-loop system is proven by using Lyapunov arguments and simulation results to confirm the efficacy of the control methodology.

Keywords

Adaptive Control; Non-Symmetric Deadzone; Hard Nonlinearity

Introduction

In many systems, performance is greatly compromised by the presence of hard nonlinearities such as friction, deadzones and backlash. An essential task in designing controllers is to cope with the undesirable effects of such nonlinearities. One of the major problems that arise in using gears is the problem of dead zone caused by the fact that gears do not mesh exactly leaving some spacing between gears teeth. During motion reversal, the spacing causes the load gear to momentarily lose contact with the driving gear, hence, tracking error occurs. The elimination dead zone can be achieved by either tightly meshing gears, which inherently increases friction and may lead to jamming, or by using very precise gears, which, in many cases, is cost prohibitive. The deadzone problem is present in conventional D.C. motors and could affect the accuracy of high precision applications. Other examples where the dead zone effect becomes problematic are evident in applications requiring high precision which is tied with utmost importance such as medical robots, semiconductor manufacturing, and precision machine tools. Deadzone in actuators, such as hydraulic servo-valves, give rise to limit cycling and instability. Many electronic devices such as diodes require a threshold voltage before it conducts current resulting in a dead zone like behaviour in its V-I characteristics. If deadzones in such systems are not accounted for, they will result in the degradation of system performance and further reduce positioning accuracy and may even destabilize such systems.

A novel concept using adaptive nonlinear control for a system containing a deadzone was investigated by (D. Recker, 1991.) The proposed method relied on the fact that by inserting a deadzone inverter before the input of the deadzone nonlinearity would negate the deadzone effect. In recent years, many researchers addressed the deadzone problem with encouraging results (N. Ahmad, 2011.) The approach in (N. Ahmad, 2011) is proved to be effective in adaptively predicting the deadzone parameters then used to construct an adaptive deadzone inverter and forcing the system to track a desired reference model. The majority of solutions employ adaptive nonlinear techniques. An abundance of algorithms utilizing Fuzzy logic or neural network was proposed in (P. Yu, 2007), (J. Campus, 1999), and (F. Lewis, 1999) to name a few. In (W. Zhonghua, 2006), an adaptive sliding mode control scheme was used to offset for a non-symmetrical deadzone nonlinearity in continuous time. The problem of chattering inherent with sliding mode control is handled by allowing a small controlled tracking error.

In this paper, and motivated by the success made in (N. Ahmad, 2011) in handling the symmetric deadzone problem, a new continuous time adaptive non-symmetrical dead zone inverter (ANSDI). The new ANSDI inverter is simple and can easily be used in conjunction with any conventional control method to reduce the effect of deadzone on the input of the

plant. In addition, the fact that the new deadzone inverter is designed in continuous time, making it amenable for combination with even more sophisticated controller or techniques such as back stepping. For the case of an uncertain deadzone parameters, an adaptive control law has been derived using Lyapunov method along with a continuous switching logic to identify the region of operation on the deadzone function. The approach is focussed on handling the deleterious effect of the deadzone region separately while it is allowed to be combined with any type of robust adaptive control schemes to ensure the overall system stability and boundedness of the states. To demonstrate the efficacy of the proposed scheme simulations utilizing a DC motor with input deadzone is presented and special emphasis is given to the stability and boundedness of the states in the system.
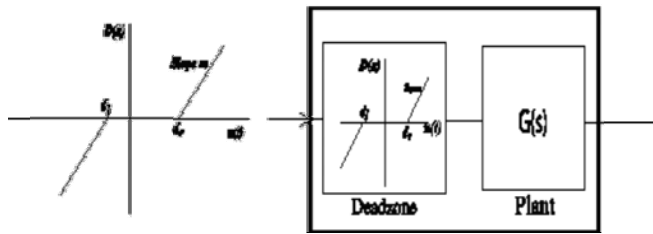

FIG. 1 (A) NON-SYMMETRIC DEAD ZONE NONLINEARITY AS A FUNCTION OF CONTROL SIGNAL. (B) INPUT DEADZONE FOR A LINEAR PLANT.

A Non-Symmetrical Deadzone Nonlinearity and Its New Continuous Time Inverse

A common representation of a nonsymmetrical deadzone nonlinearity, shown in figure 1(a), can be described as follows

$$DZ\big(u(t)\big) = \begin{pmatrix} m(u - d_r), & if & u > d_r \\ 0, & if & d_l < u < d_r \\ m(u - d_l), & if & u < d_l \end{pmatrix}, \quad (1)$$

where $DZ\big(u(t)\big)$ denotes the input of dead zone function preceding a plant input, m is the slope of the lines, $(d_r - d_l)$ is the width of the deadzone distance, and u is the input of the deadzone block as shown in figure1(b). Although the width of the deadzone spacing is assumed not to be exactly known, an upper bounds on it is given by

$$|d_r - d_l| \leq d_M \quad (2)$$

Where d_M is positive scalar.

The non-symmetrical deadzone can also be written as

$$DZ(u) = u - sat_d(u) \quad (3)$$

where $sat_d(u)$ represents a non-symmetrical saturation function and is defined as

$$sat_d(t) = \begin{pmatrix} d_r & if & u > 0 \\ u, & if & d_l < u < d_r \\ d_l, & if & u < d_l \end{pmatrix}, \quad (4)$$
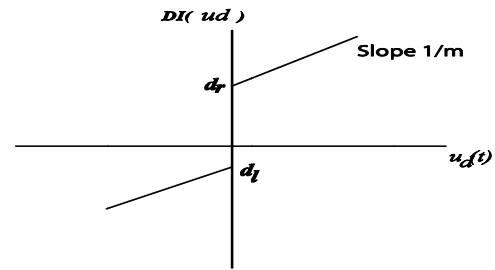

FIG. 2 THE IDEAL DEADZONE INVERSE MODEL.

Extending approach presented in (N. Ahmad, 2011), whereby compensation for the effect of the symmetrical deadzone has been achieved by adding a pre-shaping block before the input of the deadzone nonlinearity for the purpose of eliminating the effect of the dead zone and minimizing the loss of information caused by the deadzone. The dynamics of the ideal non-symmetrical deadzone inverse pre-compensator is as follows

$$u(t) = u_d(t) + \chi_l d_l + \chi_r d_r \quad (5)$$

Where u_d is the desired control law to be used in conjunction with the dynamics of the inverse compensator $\chi_l d_l$ and $\chi_r d_r$. It is noteworthy to mention that since u_d precedes the deadzone block, it is available and can be adjusted as desired. Meanwhile, χ_l and χ_r are logical indicator functions defined as follows

$$\chi_r = \begin{pmatrix} 1 & if \ u_d > 0 \\ 0 & otherwise \end{pmatrix} \quad (6)$$

$$\chi_r = \begin{pmatrix} 1 & if \ u_d < 0 \\ 0 & otherwise \end{pmatrix} \quad (7)$$

Utilizing the following function

$$sgn(u_d) \begin{pmatrix} 1 & u_d > 0 \\ -1 & u_d \leq 0 \end{pmatrix} \quad (8)$$

makes (6) and (7)

$$\chi_r = \frac{1 + sgn(u_d)}{2} \quad (9)$$

$$\chi_l = \frac{1 - sgn(u_d)}{2} \quad (10)$$

Equations (9) and (10) are not smooth functions which makes them non-differentiable; hence, they can not be combined with techniques requiring differentiability of control laws such as backstepping. Therefore, a continuous time representation of χ_l and χ_r can be devised by specifying a smoothly switching dynamics. The dynamics of the switching function may be written as

$$\dot{\Phi}(t) = u_d - \sigma|u_d|\Phi(t), \quad (11)$$

where σ is a positive constant. The equilibrium points for $\sigma\Phi$ is obtained by setting $\dot{\Phi}(t) = 0$ which results in

$$\sigma\Phi = \frac{u_d}{|u_d|} = sgn(u_d). \qquad (12)$$

Hence, utilizing the term $(\sigma\Phi)$ to construct

$$\chi_r = \frac{1+\sigma\Phi(t)}{2} \qquad (13)$$

$$\chi_l = \frac{1-\sigma\Phi(t)}{2} \qquad (14)$$

gives a continuous time implementation for the logical operators χ_r and χ_l.

Claim: The dynamics of $\Phi(t)$ are bounded input bounded output (BIBO) stable. *Proof*: Let $V_\Phi = \frac{\Phi^2}{2}$, Then the time derivative of V_Φ is given by

$$\begin{aligned} \dot{V}_\Phi &= \Phi(u_d - \sigma|u_d|\Phi) \\ &\leq -|u_d||\Phi|(\sigma|\Phi| - |sgn(u_d)sgn(\Phi)|) \\ &\leq -|u_d||\Phi|(\sigma\Phi - 1) \end{aligned} \qquad (15)$$

showing that $\dot{V}_\Phi < 0$ as long as $\Phi \geq \frac{1}{\sigma}$. Hence, all solutions of $\Phi(t)$ are bounded by a ball of radius $\frac{1}{\sigma}$ which can be made arbitrarily small by proper choice of - σ.

Adaptive Control of DC Motor with Unknown Non-Symmetrical Deadzone

Whenever the exact deadzone parameters $d = [d_r^* \, d_l^*]$ are not known a priori, estimates defined as $\hat{d} = [\hat{d}_r \hat{d}_l]$ will be used along with an adaptive update law to adjust them appropriately.

Hence, the combined functional representation of the adaptive inverse function and the deadzone nonlinearity can be written as

$$u(t) = u_d + \chi_r \widehat{d_r} + \chi_l \widehat{d_l} \qquad (16)$$

The application of the designed pre-compensator dynamics to the deadzone function gives

$$\begin{aligned} u(t) = u_d &+ \chi_r \widehat{d_r} + \chi_l \widehat{d_l} + \\ &+ sat(u_d + \chi_r \widehat{d_r} + \chi_l \widehat{d_l}) \end{aligned} \qquad (17)$$

By defining the deadzone error parameters as

$$\tilde{d}^T = \begin{bmatrix} d_r^* - \widehat{d_r} \\ d_l^* - \widehat{d_l} \end{bmatrix} \qquad (18)$$

$$\tilde{\chi} = [\chi_r \chi_l] \qquad (19)$$

equation (17) can be written as

$$u(t) = u_d - \tilde{d}^T \tilde{\chi} + \tilde{d}^T \delta \qquad (20)$$

where δ, is the modelling mismatch between the ideal exact deadzone inverse pre-compensator and the adaptive one, is shown to be bounded as $(\delta < 1)$ in (F. Lewis, 1999). For the case where the deadzone spacing

parameter is unknown, then an adaptive non-symmetric deadzone inverse function can be combined with any standard linear controller, such as PD, to offset the effect of the input deadzone nonlinearity. The dynamics of second order system such as a DC servo system can be written as

$$J\ddot{\theta} + \beta\dot{\theta} + T_d = T, \qquad (21)$$

where J is the inertia, β is the coefficient of viscous friction, and T_d represents the external disturbance caused by variable load. For the purpose of emphasizing the performance of the deadzone inverse on minimizing the effect of the deadzone on the system, it is assumed that the inertia is known and constant. Therefore, without any loss of generality, one degree of freedom (DOF) which could represent a DC motor withoutload or friction variations, and suffers from input deadzone can be written as

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= T = D(u) = Deadzone(u) \end{aligned} \qquad (22)$$
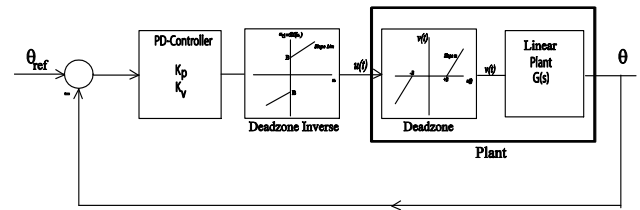

FIG. 3 PROPOSED SYSTEM BLOCK DIAGRAM

The internal control signal $D(u)$ is assumed to be unmeasurable. The controller consists of a feedforward portion captured by the term ($\tilde{d}^T\tilde{\chi}$), and a feedback desired control law represented by $u_d(t)$. The following control law is proposed for the system (22)

$$u(t) = u_d - \tilde{d}^T\tilde{\chi} + \tilde{d}^T\delta \qquad (23)$$

$$u_d(t) = -k_p\tilde{\theta} - k_v\tilde{\omega} + \dot{\omega}_{ref} \qquad (24)$$

where θ_{ref} and ω_{ref} are the desired tracking trajectory and velocity respectively; $\tilde{\theta} = (\theta - \theta_{ref})$ and $\tilde{\omega} = (\omega - \omega_{ref})$ are the desired trajectory tracking errors. Moreover, $\tilde{d}$ is the adaptation terms used to adjust $\hat{d}$ to match the actual deadzone parameters d^* by the following adaptation law

$$\dot{\hat{d}} = \Gamma\bar{\chi}(\tilde{\theta} + \tilde{\omega}) - \varsigma\Gamma\hat{d}|\tilde{\theta} + \tilde{\omega}| \qquad (25)$$

where Γ is a diagonal invertible matrix, ς is a positive constant, and $\bar{\chi}$ is given by equation (19). The inverse deadzone dynamics are given by

$$\dot{\Phi}(t) = u_d - \sigma|u_d|\Phi(t) \qquad (26)$$

Claim. The control law (23) for the system defined by (22) will ensure the closed-loop stability and boundedness of tracking error and hence reducing the effects of deadzone on the control law and system output.

Proof. Using the following positive definite control Lyapunov function

$$V = \frac{1}{2}(k_p + k_v)\tilde{\theta}^2 + \tilde{\theta}\tilde{\omega} + \frac{1}{2}\tilde{\omega}^2 + \frac{1}{2}\tilde{d}^T\Gamma^{-1}\tilde{d} \quad (27)$$

where $k_p > 0$, $k_v > 1$, $\tilde{\theta} = (\theta - \theta_{ref})$, and $\tilde{\omega} = (\omega - \omega_{ref})$. Differentiating along the trajectories of the system yields

$$\begin{aligned}\dot{V} &= (k_p + K_v)\tilde{\theta}\tilde{\omega} + \tilde{\omega}^2 + (\tilde{\theta} + \tilde{\omega})\dot{\tilde{\omega}} + \tilde{d}^T\Gamma^{-1}\dot{\tilde{d}}\\&= (k_p + K_v)\tilde{\theta}\tilde{\omega} + \tilde{\omega}^2(\tilde{\theta} + \tilde{\omega})(D(u) - \dot{\omega}_{ref})\\&\quad + \tilde{d}^T\Gamma^{-1}\dot{\tilde{d}}\\&= (k_p + K_v)\tilde{\theta}\tilde{\omega} + \tilde{\omega}^2(\tilde{\theta} + \tilde{\omega})(u_d - \tilde{d}^T\chi + \tilde{d}^T\delta - \dot{\omega}_{ref}) + \tilde{d}^T\Gamma^{-1}\dot{\tilde{d}},\end{aligned} \quad (28)$$

where $\delta = D[\widehat{DI}(u_d)] - u_d$ represents the modelling mismatch error shown to be bounded as ($\|\delta\| \le 1$)(F. Lewis,1999). Substituting the control law (24) for u_d (t) in (28), the last term becomes

$$\begin{aligned}(\tilde{\theta} + \tilde{\omega})(&-k_p\tilde{\theta} - k_v\tilde{\omega} + \dot{\omega}_{ref}\\&- \tilde{d}^T\chi + \tilde{d}^T\delta - \dot{\omega}_{ref}).\end{aligned} \quad (29)$$

Collecting terms and simplifying further results in

$$\dot{V} = -k_p\tilde{\theta}^2 - k_v\tilde{\omega}^2 - (k_p + k_v)(\tilde{\theta}\tilde{\omega}) + (\tilde{\theta} + \tilde{\omega})(-\tilde{d}^T\chi + \tilde{d}^T\delta) + \tilde{d}^T\Gamma^{-1}\dot{\tilde{d}} \quad (30)$$

Inserting back in (28)and simplifying

$$\dot{V} = -k_p\tilde{\theta}^2 - (k_v + 1)\tilde{\omega}^2 - (k_p + k_v)(\tilde{\theta}\tilde{\omega}) + (\tilde{\theta} + \tilde{\omega})(-\tilde{d}^T\chi + \tilde{d}^T\delta) + \tilde{d}^T\Gamma^{-1}\dot{\tilde{d}}. \quad (31)$$

Utilizing the adaptation law (25) the derivative of the Lyapunov function is reduced to

$$\begin{aligned}\dot{V} = &-k_p\tilde{\theta}^2 - (k_v + 1)\tilde{\omega}^2 + (\tilde{\theta} + \tilde{\omega})(-\tilde{d}^T\chi + \tilde{d}^T\delta) + \tilde{d}^T\Gamma^{-1}(\Gamma\chi(\tilde{\theta} + \tilde{\omega}) - \varsigma\Gamma\hat{d}|\tilde{\theta} + \tilde{\omega}|),\end{aligned} \quad (32)$$

then collecting terms yields

$$\begin{aligned}\dot{V} = &-k_p\tilde{\theta}^2 - (k_v + 1)\tilde{\omega}^2\\&+ (\tilde{\theta} + \tilde{\omega})(-\tilde{d}^T\chi + \tilde{d}^T\delta)\\&+ (\tilde{\theta} + \tilde{\omega})(+\tilde{d}^T\bar{\chi}) - \varsigma\tilde{d}^T\hat{d}|\tilde{\theta} + \tilde{\omega}|,\end{aligned} \quad (33)$$

which yields

$$\begin{aligned}\dot{V} = &-k_p\tilde{\theta}^2 - (k_v + 1)\tilde{\omega}^2\\&+ \tilde{d}^T[(\tilde{\theta} + \tilde{\omega})\delta - \varsigma\hat{d}|\tilde{\theta} + \tilde{\omega}|).\end{aligned} \quad (34)$$

Rearranging terms

$$\begin{aligned}\dot{V} = &-k_p\tilde{\theta}^2 - (k_v + 1)\tilde{\omega}^2\\&+ (\tilde{\theta} + \tilde{\omega})\tilde{d}^T[\delta - \varsigma\hat{d}\cdot sgn(\tilde{\theta} + \tilde{\omega})].\end{aligned} \quad (35)$$

The last term in (35)can be bounded as

$$\le \|(\tilde{\theta} + \tilde{\omega})\tilde{d}^T\| [\|\delta\| - \varsigma\|\hat{d}\cdot sgn(\tilde{\theta} + \tilde{\omega})\|],$$

where $\|\delta\| \le 1$, $\|\hat{d}\| \le d_M$, and $\|sgn(\tilde{\theta} + \tilde{\omega})\| \le 1$. In conclusion, it is ensured that $\dot{V} < 0$ as long as

$$[1 - \varsigma d_M] \le 0 \text{ or } \varsigma \ge \frac{1}{d_M}. \quad (36)$$

From inequality (36), it is clearly seen that $\dot{V} < 0$ as long as the gain constant ς chosen to be greater than the upper bounds of the deadzone spacing d_M. As a result, the states of the system are stable, bounded, and asymptotic tracking is achieved.

Simulations

Simulations of the system in (22) under the adaptive control law (23) and (24) have been performed for a sinusoidal reference trajectory given by $\theta_d = 2 + \sin(\omega_d) + \frac{1}{2}\sin(5\omega_d)$, shown in figure 4, with adaptation law (25). The upper bounds on actuator actual spacing $d_M = d_r - d_l = 15$ is assumed unknown. Meanwhile, the initial values of $\hat{d}_l$ and $\hat{d}_r$ are set to be zero and no apriori knowledge of their values are needed. The exact values of the simulated dead zone parameters spacings are $d_r^* = 5.0$ and $d_l^* = -10.0$. The adaptively compensated position tracking error $\tilde{\theta}$ is shown in figure 4; meanwhile, the tracking errors of the system under a PD-controller only are shown in dashed line. A reasonably good tracking performance without steady state error is achieved.

TABLE 1 PARAMETERS UTILIZED IN THE EXAMPLE

	Systems Physical Attributes		
	Parameter	Value	Unit
1	k_p	200	Gain constant
2	k_v	20	Gain constant
3	d_r^*	5.0	N.m
4	d_l^*	-10	N.m
5	σ	1.0	N.m/rad
6	Γ	$\begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$	Gains
7	J	1	$\frac{V}{rad}\cdot s^{-2}$
8	ς	0.01	Gain

The efficacy of the proposed control designs is tested for a non-symmetrical deadzone spacing preceding

the linear part of a second order plant simulating a DC motor. In figure 6, the control effort error due to mismatch between the non-symmetrical deadzone and the adaptive deadzone inverse, $\delta = DZ[\widehat{DI}(u_d)] - u_d$, asymptotically approaches zero in tune with the development of the adaptations parameters $\hat{d}_r$ and $\hat{d}_l$ shown in figures 7 and 8 . Figure 9 shows the control effort u (t) for the dead zone compensated system $DZ[\widehat{DI}(u_d)]$, which demonstrates the elimination of the distortion caused by the deadzone as opposed to the inability of the PD controller.

FIG. 7 EVOLUTION OF THE ADAPTATION $\hat{d}_l$ APPROACHES THE ACTUAL VALUE OF $d_l^* = 10$
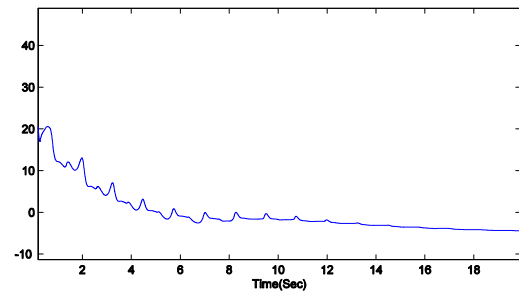
FIG. 8 EVOLUTION OF THE ADAPTATION $\hat{d}_r$ APPROACHES THE ACTUAL VALUE OF $d_r^* = 5$

FIG. 4 THE MOTOR ANGLE $\Theta(T)$ TRACKING OF θ_{ref}=2+sin (π t)+$\frac{1}{2}$ sin ($5 \pi t$).
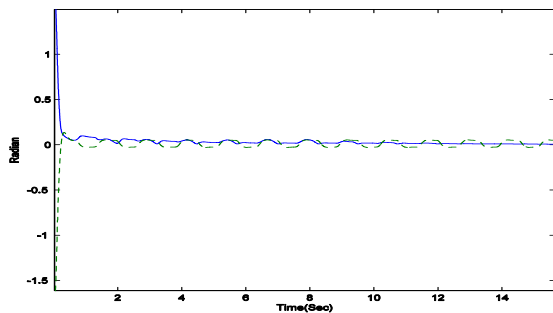

FIG. 5 THE ADAPTIVELY COMPENSATED POSITION TRACKING ERROR $(\Theta)\check{}$(SOLID) VS. THE TRACKING ERROR OF THE SYSTEM UNDER A PD-CONTROLLER (DASHED.)
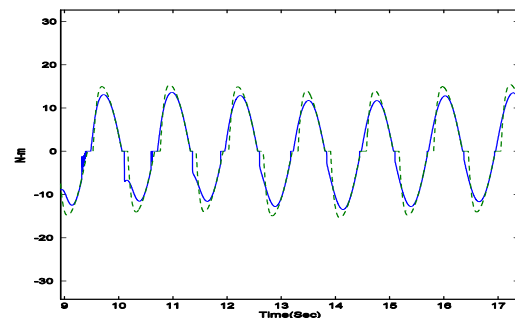
FIG. 9 THE CONTROL EFFORT U(T) FOR THE DEADZONE COMPENSATED SYSTEM $dz[\widehat{DI}(\mathbf{u}_d)]$, WHICH DEMONSTRATES THE ELIMINATION OF THE DISTORTION CAUSED BY THE DEADZONE AS OPPOSED TO THE INABILITY OF THE PD CONTROLLER (DASHED).

Conclusions

An adaptive inverse deadzone combined easily with any conventional control methods such as a PD controller has been shown to not only effectively stabilize a second order system, but also achieve asymptotic tracking. The deadzone inverse model is non-symmetric and implemented in continuous time. The adaptive deadzone inverse controller is smoothly differentiable and can easily be combined with any of the advanced control methodologies. The asymptotic stability of the closed-loop system has been proven by using Lyapunov arguments and simulation results confirmed the efficacy of the control methodology.
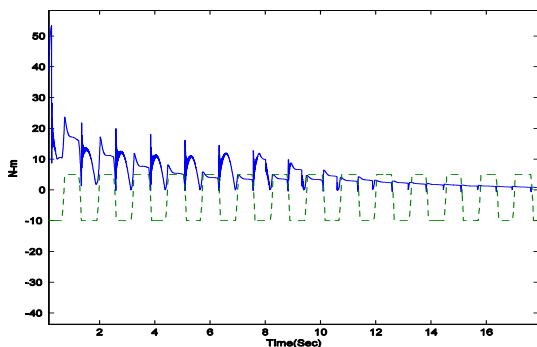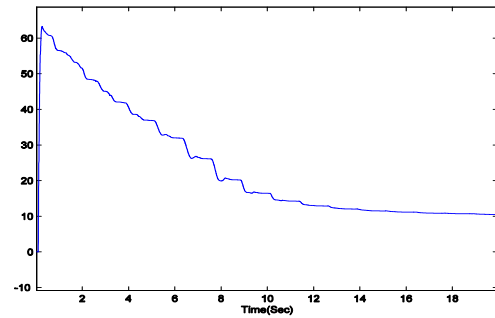

FIG.6 CONTROL EFFORT ERROR REDUCTION $\delta = D[\widehat{DI}(u_d)] - u_d$(SOLID) VS.THEERROR FOR A PD COMPENSATED CONTROL SIGNAL $D(u_d) - u_d$(DASHED).

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